

Medium-induced jet evolution: multiple branching and thermalization

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Abstract

For an energetic jet propagating through a weakly-coupled quark-gluon plasma, we present the physical picture of jet quenching in longitudinal phase-space, as emerging from the interplay between the medium-induced, quasi-democratic, branchings and the elastic collisions responsible for the thermalization of the soft branching products.

1. Introduction

The experimental studies of jets in Pb+Pb collisions at the LHC have triggered intense theoretical efforts towards understanding global observables like the shape of a jet propagating through a dense QCD medium and the energy transfer by the jet towards the medium. These efforts lead to the emergence of a new picture for the in-medium jet evolution, in which the energy of the hard components of the jet is efficiently transmitted, via multiple, quasi-democratic, branchings, to a multitude of comparatively soft gluons, which are then deviated towards large angles by rescattering in the medium [1, 2, 3, 4, 5, 6, 7]. In particular, Ref. [6] presented for the first time the picture of this evolution in longitudinal phase-space, with the longitudinal axis defined as the direction of propagation of the leading particle. This picture will be succinctly summarized in what follows.

2. Kinetic theory for jet evolution

We study the parton distribution produced by a high-energy jet propagating through a weakly-coupled quark-gluon plasma in thermal equilibrium at temperature T . The leading particle (LP) which initiates the jet has a high energy $E \gg T$ and a comparatively small virtuality. We concentrate on the *medium-induced* evolution,

as triggered by the collisions between the partons from the jet and the constituents of the medium. One can distinguish between two types of collisions:

- (i) elastic, $2 \rightarrow 2$, collisions, which entail energy and momentum transfer between the jet and the medium;
- (ii) inelastic collisions, like $2 \rightarrow 3$ or, more generally, $1+(\text{many}) \rightarrow 2+(\text{many})$, in which a parton from the jet undergoes a $1 \rightarrow 2$ branching.

At weak coupling, such processes can be described by a kinetic equation for the gluon distribution [8, 9],

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f(t, \mathbf{x}, \mathbf{p}) = C_{\text{el}}[f] + C_{\text{br}}[f]. \quad (1)$$

Here, $f(t, \mathbf{x}, \mathbf{p})$ is the gluon occupation number, $\mathbf{v} = \mathbf{p}/p$ with $p \equiv |\mathbf{p}|$ is the gluon velocity, and $C_{\text{el}}[f]$ and $C_{\text{br}}[f]$ are collision terms which encompass the elastic and inelastic processes, respectively. Explicit expressions can be found in [9, 10, 11], but the general equation is too complicated to be solved in practice, even numerically. In what follows, we shall examine the relevant processes in more detail, in order to motivate a simple approximation to Eq. (1) which is tractable in practice [6].

The medium-induced parton cascades are controlled by relatively hard gluons, with momenta p within the range $T \ll p \ll E$, which undergo small-angle scattering. The elastic collision term can therefore be evaluated in the Fokker-Planck approximation [10, 11]:

$$C_{\text{el}}[f] \simeq \frac{1}{4} \hat{q} \nabla_p \cdot \left[\left(\nabla_p + \frac{\mathbf{v}}{T} \right) f \right], \quad (2)$$

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with $\hat{q} \sim \alpha_s^2 T^3 \ln(1/\alpha_s)$ the jet quenching parameter.

Consider a test particle which at $t = 0$ has a high momentum $p_0 \gg T$ oriented along the z -axis. Eq. (2) implies that this particle loses energy according to $\langle p_z(t) \rangle \simeq p_0 - \eta t$, with $\eta \equiv \hat{q}/4T$ (the ‘drag coefficient’), and at the same time suffers transverse and longitudinal momentum broadening: $\langle p_\perp^2 \rangle \simeq \hat{q}t$ and $\langle \Delta p_z^2 \rangle \simeq (\hat{q}/2)t$. This dynamics eventually drives the test particles to thermal equilibrium. Indeed, one can easily check that Eq. (2) admits the Maxwell-Boltzmann thermal distribution $f_{\text{eq}} \propto e^{-p/T}$ as a fixed point.

In the absence of inelastic process, the test particle would lose its initial energy p_0 via drag after a time $t_{\text{drag}}(p_0) \simeq p_0/\eta$ and then thermalize via diffusive processes under an additional time $\Delta t \sim t_{\text{rel}}$, with

$$t_{\text{rel}} \equiv \frac{4T^2}{\hat{q}} \sim \frac{1}{\alpha_s^2 T \ln(1/\alpha_s)}. \quad (3)$$

Observing that $t_{\text{drag}}(p_0) \simeq (p_0/T)t_{\text{rel}} \gg t_{\text{rel}}$ when $p_0 \gg T$, it becomes clear that the overall duration of the thermalization process is controlled by the first stage — the energy loss via drag.

But after adding the inelastic collisions, i.e. the term $C_{\text{br}}[f]$ in the r.h.s. of (1), the above scenario changes dramatically, at least for the relatively hard components of the jet with $p \gg T$. This is so since multiple branching is much more efficient than elastic collisions in redistributing the energy among the soft modes.

Indeed, in the presence of inelastic collisions, a gluon with momentum $p \gg T$ has only a finite lifetime,

$$t_{\text{br}}(p) = \frac{1}{\alpha_s} \sqrt{\frac{p}{\hat{q}}}, \quad (4)$$

until it disappears via a *democratic branching*, i.e. until it splits into a pair of gluons which carry comparable fractions of the energy p of their parent gluon [8, 12, 2]. The emergence of this scale can be understood as follows: the probability $\Delta\mathcal{P}$ for a branching to occur during an interval Δt can be estimated as [13, 14]

$$\Delta\mathcal{P} \simeq \alpha_s \frac{\Delta t}{t_{\text{f}}} \simeq \alpha_s \sqrt{\frac{\hat{q}}{p}} \Delta t, \quad (5)$$

where $t_{\text{f}} \simeq p_z/p_\perp^2$ is the ‘formation time’, i.e. the quantum-mechanical duration of a branching process where the softest emitted gluon has 3-momentum $\mathbf{p} = (p_\perp, p_z)$ with $p_z \simeq p \gg p_\perp$. For a branching occurring in the vacuum, p_z and p_\perp are independent variables, but in the presence of the medium, the emitted gluon acquires a transverse momentum $p_\perp^2 \simeq \hat{q}t_{\text{f}}$ during the formation time. Using these two relations, $t_{\text{f}} \simeq p/p_\perp^2$ and

$p_\perp^2 \simeq \hat{q}t_{\text{f}}$, one deduces $t_{\text{f}}(p) \simeq \sqrt{p/\hat{q}}$, which explains the second equality in (5). This probability $\Delta\mathcal{P}$ become of order one after a time $\Delta t \simeq t_{\text{br}}(p)$, cf. Eq. (4).

Using $\hat{q} \sim \alpha_s^2 T^3 \ln(1/\alpha_s)$, it is easy to check that

$$t_{\text{drag}}(p) \gg t_{\text{br}}(p) \gg t_{\text{rel}} \quad \text{for} \quad p \gg T. \quad (6)$$

Hence, a hard gluon with $p_0 \gg T$ disappears via democratic branchings before having the time to lose a substantial fraction of its energy via drag. In turn, the daughter gluons will split again and again, thus eventually producing a gluon cascade. Each new gluon generation in this cascade has a lower energy and hence a shorter lifetime than the previous ones. Accordingly, the overall lifetime of the cascade is of the order of the branching time $t_{\text{br}}(p_0)$ of the initial gluon.

The cascade stops when the branching products become as soft as the medium constituents: $p \sim T$. Indeed, the soft gluons from the jet can efficiently thermalize via elastic collisions, over a time interval t_{rel} which is comparable with the corresponding branching time: $t_{\text{br}}(T) \sim t_{\text{rel}}$. In thermal equilibrium, the detailed balance principle ensures that splitting ($1 \rightarrow 2$) and recombination ($2 \rightarrow 1$) processes exactly compensate each other, so the inelastic collision term vanishes, so like the elastic one: $C_{\text{el}}[f] = C_{\text{br}}[f] = 0$ for $f = f_{\text{eq}}$.

Via thermalization, the whole energy p_0 of the initial gluon is ultimately transmitted to the medium. As anticipated, the characteristic time for thermalization is fixed by the branching dynamics and is of order $t_{\text{br}}(p_0)$. This time is much shorter (when $p_0 \gg T$) than the collisional time scale $t_{\text{drag}}(p_0)$ — the would-be thermalization time in the absence of branchings.

So far, we have implicitly assumed that the lifetime $t_{\text{br}}(p_0)$ of the cascade is smaller than the size L of the medium which is available to the jet along the longitudinal (z) axis. Together with (4), this condition implies an upper limit on the momentum p_0 of the primary gluon: $p_0 \lesssim \omega_{\text{br}}(L) \equiv \alpha_s^2 \hat{q} L^2$. For the conditions at the LHC, it turns out that this medium scale $\omega_{\text{br}}(L)$ is only moderately hard: using typical values like $\hat{q} = 1 \text{ GeV}^2/\text{fm}$, $L = 5 \text{ fm}$, and $\alpha_s = 0.3$, one finds $\omega_{\text{br}}(L) \simeq 12 \text{ GeV}$. This is smaller than the energy $E \gtrsim 100 \text{ GeV}$ of the LP, but larger than the medium temperature $T \lesssim 1 \text{ GeV}$.

The fact that $E \gg \omega_{\text{br}}(L)$ implies that the LP cannot disappear inside the medium: it rather emerges in the final state, with a reduced energy though. On the other hand, the LP can abundantly emit relatively soft primary gluons, with momenta $T \ll p_0 \lesssim \omega_{\text{br}}(L)$, which then generate *mini-jets* via democratic branchings. The energy carried by these mini-jets is eventually transmitted to the medium, as already explained. The total energy

lost by the jet in this way can be estimated as [2, 3]

$$\Delta E_{\text{therm}} \simeq \nu \omega_{\text{br}}(L) = \nu \alpha_s^2 \hat{q} L^2, \quad (7)$$

where $\nu \simeq 2.5$ is the average number of primary gluons with the hardest possible energy, i.e. $p_0 = \omega_{\text{br}}(L)$.

During most stages of the branching process, the cascade is built with relatively hard gluons, which are nearly collinear with the LP: $p_z \gg p_\perp$. It therefore makes sense to focus on the *longitudinal* dynamics, as obtained after integrating out the transverse phase-space. This motivates the following, relatively simple, kinetic equation, for the *longitudinal gluon distribution* $f_\ell(t, z, p_z) \equiv \int d^2 \mathbf{x}_\perp d^2 \mathbf{p}_\perp f(t, \mathbf{x}, \mathbf{p})$ [6]:

$$(\partial_t + \nu \partial_z) f_\ell(t, z, p) = \frac{\hat{q}}{4} \partial_p \left[\left(\partial_p + \frac{\nu}{T} \right) f_\ell(t, z, p) \right] \quad (8)$$

$$+ \frac{1}{t_{\text{br}}(p)} \int_r dx \mathcal{K}(x) \left[\frac{1}{\sqrt{x}} f_\ell \left(t, z, \frac{p}{x} \right) - \frac{1}{2} f_\ell(t, z, p) \right].$$

Here, $p \equiv p_z$, $\nu \equiv p/|p|$, x is the splitting fraction, with $0 \leq x \leq 1$, $\mathcal{K}(x) \equiv [1 - x(1-x)]^{\frac{5}{2}} / [x(1-x)]^{\frac{3}{2}}$ is the BDMPSS kernel [13, 14], and the subscript r on the integral over x means that the branching process is cut off at the soft scale $p = T$. The two terms within the inelastic collision integral are recognized as the *gain* term and *loss* term, respectively. The initial condition reads

$$f_\ell(t=0, z, p) = \delta(p - E) \delta(z), \quad (9)$$

corresponding to a LP with longitudinal momentum $p = E$ which enters the medium at $t = 0$ and $z = 0$.

3. The longitudinal gluon distribution

In this section, we shall present some of the results obtained in [6] via analytic and numerical studies of Eq. (8). Before we address the general equation (8), we consider a simpler, related, problem, where the inelastic collision term in the r.h.s. of (8) is replaced by a steady source for particles with longitudinal momentum $p_0 > T$ which propagate at the speed of light:

$$C_{\text{br}}[f_\ell] \rightarrow T \delta(p - p_0) \delta(z - t). \quad (10)$$

With $p_0 \sim T$, this source mimics the effects of the branching term in so far as the distribution of the soft gluons ($|p| \lesssim T$) is concerned. This source problem turns out to be exactly solvable [6], with the result illustrated in Fig. 1. One sees a two component structure, with a front and a tail. The *front* is made with relatively hard gluons, with momenta $T \lesssim p \lesssim p_0$, which

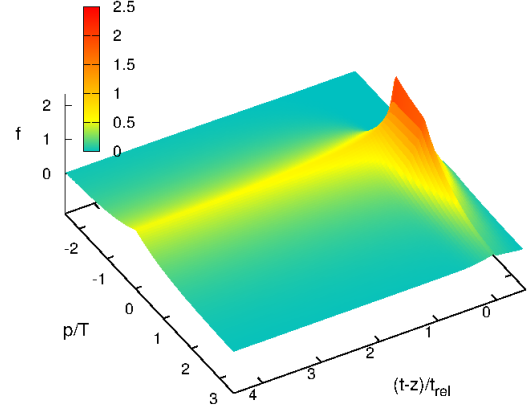


Figure 1: The longitudinal distribution produced by the steady source (10) shown as a function of $t - z$ and p , for $p_0 = T$.

propagate at the speed of light: $z = t$. These are particles injected by the source which have only partially lost their energy via drag. The *tail* lies behind the front ($z < t$) and is made with particles which have thermalized under the combined effect of drag and diffusion. The distribution far behind the front is simply the (one-dimensional) thermal distribution:

$$f_\ell(t, z, p) \simeq \frac{1}{2} e^{-|p|/T} \quad \text{when } t - z \gtrsim t_{\text{rel}}. \quad (11)$$

We now turn to the general equation (8) with the initial condition (9). The time scales inherent in this equation are the branching time $t_{\text{br}}(E)$ for the LP and the relaxation time t_{rel} via elastic collisions. In the experimental situation at the LHC, one has $L < t_{\text{br}}(E)$, as already mentioned, hence the LP is expected to survive in the final state. This is indeed visible in the numerical results displayed in Fig. 2, as numerically obtained for $E = 90 T$ and $t_{\text{br}}(E) \simeq 10 t_{\text{rel}}$ [6].

Fig. 2.a shows the distribution at the very early time $t = 0.1 t_{\text{br}}(E)$, when most of the energy is still carried by the LP. Hence the energy distribution $|p| f_\ell / T$ shows a pronounced peak at $p/T = 90$ and at $z = t$. Yet, this peak shows some spreading in p , as a consequence of early emissions, which are necessarily soft: the typical quanta emitted up to time t have $p \lesssim \omega_{\text{br}}(t) = \alpha_s^2 \hat{q} t^2$.

At the larger time $t = 0.3 t_{\text{br}}(E)$, cf. Fig. 2.b, the softening of the distribution in p is clearly visible, albeit a pronounced LP peak still exists. One can now distinguish the characteristic ‘front’ + ‘tail’ structure anticipated in Fig. 1. The front at $z = t$ involves relatively hard gluons, whose momentum distribution (within the

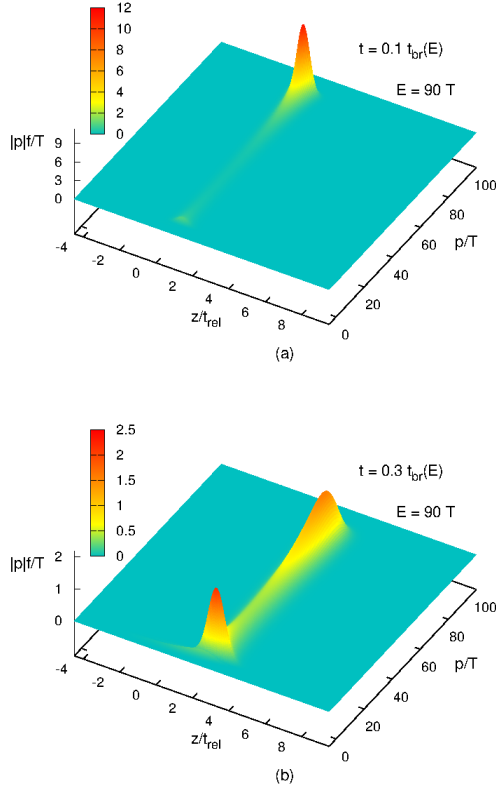


Figure 2: The phase-space energy distribution $|p|f_t/T$ produced by an energetic jet with $E = 90 T$ at two successive times: (a) an early time $t = 0.1 t_{br}(E)$, when the jet is almost unquenched; (b) a larger time $t = 0.3 t_{br}(E)$, when the jet is partially quenched.

range $T < p \ll E$) is given by the scaling spectrum $f_t \propto 1/p^{3/2}$, as expected for quasi-democratic branchings [8, 2]. This scaling behavior, which is a signature of *wave turbulence* [2, 3], is better visible in Fig. 3, which shows the function $(p/T)^{3/2} f_t$ at $z = t$. Similar results have been obtained in a kinetic theory study of the thermalization of the quark-gluon plasma [15].

The front in Fig. 2.b also shows a secondary peak at $p = T$, due to the accumulation of gluons at the lower end of the cascade. Such gluons are abundantly produced via branchings and they cannot thermalize instantaneously — rather, they need a time $\sim t_{rel}$ to that aim. Yet, since $t = 0.3 t_{br}(E) \simeq 3 t_{rel}$ is relatively large compared to t_{rel} , a thermalized tail at $z \lesssim t - t_{rel}$ develops indeed, as visible too in Fig. 2.b. This tail carries the energy lost by the jet towards the medium. The numerical studies demonstrate that this energy loss grows with time like t^2 , in agreement with Eq. (7) [6].

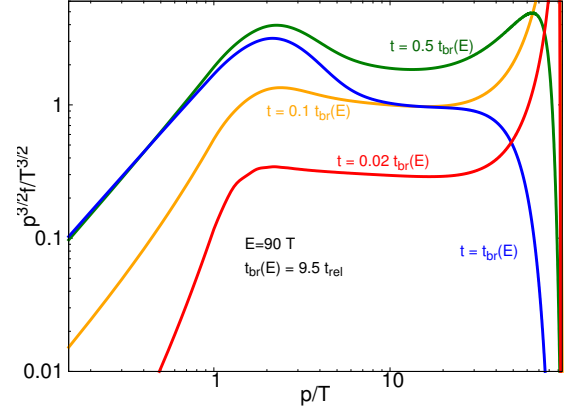


Figure 3: The gluon distribution in p at $t = z$ for $E = 90 T$ and for 4 values of time. The figures show a rather broad window of approximate scaling behavior, $f_t \propto 1/p^{3/2}$, at not too large times $t \lesssim t_{br}(E)$.

These results allow for a qualitative comparison with the phenomenology of di-jet asymmetry at the LHC [16, 17]. (For a more quantitative comparison, one could take $T = 0.5 \text{ GeV}$ and $t_{rel} = 1 \text{ fm}$.) The early situation in Fig. 2.a, where the jet is essentially unquenched, is illustrative for the leading jet, which crosses at most a very narrow slab of matter. The situation in Fig. 2.b, where the jet looks partially quenched, is representative for the subleading jet in a di-jet event characterized by a large asymmetry. For even larger times, $t \gtrsim t_{br}(E)$, both the LP and the front would disappear and the whole energy would be found in the thermalized tail [6].

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